2012 IMSA Junior High Mathematics Competition 7th Individual

1. A math store buys calculators for $100 each. After the store marks up the price by 70% and offers a 30% discount on the markup price, how much (in dollars) does the customer pay before paying the 8.2% tax?

2. If half the perimeter of a heptagon is 31.5 cm, what is the perimeter (in centimeters) of a pentagon with sides lengths equal to those of the heptagon?

3. An alien IMSA student calculates that there are 7 scondies to a mogel, 2 mogels to 3 meytons, 4 meytons to a nince, and 12 ninces to 5 vastkos. How many scondies are equal to 13 vatskos? Express your answer in decimal form.

4. A whole number plus that number squared plus that number cubed minus 97 is -13. What is that number?

5. What is the value of $1 - 3 + 5 - 7 + 9 - 11 + \cdots + 97 - 99$?

6. There are 27 IMSA students in an IMSA math class. Out of them, 18 students can answer every question on the seventh-grade individual math contest, 13 can answer every question on the eighth-grade individual math contest, and 7 can answer every question on both math contests. How many students in the class cannot answer every question on either contest?

7. If an equilateral triangle has perimeter 6, what is its area?

8. If 6 cats can eat 6 mice in 6 hours, then how many mice can 1 cat eat in 12 hours?

9. A rectangular patch of grass is x feet by (x + 10) feet. A walkway that is 3 feet wide is placed completely around the pool forming a larger rectangle. The area of the rectangle formed by the patch of grass and walkway is 192 square feet greater than the area of the grass patch alone. What is the area of the grass patch, in square feet?

10. Find the greatest ten-digit positive multiple of 12 using each digit once and only once.

11. What is the last digit (ones) of the product of the positive prime numbers less than 100?

12. How many squares are on a traditional 8×8 checkerboard if squares can only be made using only the edges of the individual 1×1 squares?

13. A point is randomly selected within the rectangle defined by the vertices: (0,0), (5,0), (5, 7), and (0,7). What is the probability that the x-coordinate of the point is less than the y-coordinate? Express your answer as a common fraction reduced to lowest terms.

14. Using only the digits 1 through 5, how many even three-digit positive integers less than 500 can be written if each digit can be used more than once?

15. How many numbers belong to the sequence: $17, 24, 31, 38, \ldots\ 2012$, where the difference between consecutive numbers is 7?

16. Scott was born on a Tuesday. What is the probability that exactly two of his three best friends were also born on Tuesday? Express your answer as a common fraction reduced to lowest terms.

17. A collection of nickels, dimes and pennies has an average value of 7 cents per coin. If a nickel were replaced by five pennies, the average would drop to 6 cents per coin. How many dimes are in the collection?

18. A number is five more than the product of two consecutive positive integers and is also an integral multiple of both 55 and 121. Let the smaller of the two consecutive positive integers be represented by k. Find the sum of all distinct values of k if $k < 157$.

19. If the letters of "ILLINOIS" are mixed up, selected randomly one at a time without replacement, and written down in the order drawn, find the probability that the three "I's" are together, in adjacent positions in this random arrangement of letters. Express your answer as a common fraction reduced to lowest terms.

20. How many ways can we choose 4 elements of {1, 2…12} without replacement such that no two chosen numbers are consecutive?

2012 IMSA Junior High Mathematics Competition 8th Individual

1. What is the area of a square with side length 8?

2. Aunt Anna is 42 years old. Caitlin is 5 years younger than Brianna, and Brianna is half as old as Aunt Anna. How old is Caitlin?

3. Ara and Shea were once the same height. Since then, Shea has grown 20% while Ara has grown half as many inches as Shea. Shea is now 60 inches tall. How tall, in inches, is Ara now?

4. The number 64 has the property that it is divisible by its units digit. How many whole numbers *strictly* between 10 and 40 have this property?

5. If 20% of a number is 12, what is 50% of the same number?

6. A group of children riding on bicycles and tricycles rode past Billy Bob's house. Billy Bob counted 7 children and 19 wheels. How many tricycles were there?

7. Blake and Jenny each took four 100-point tests. Blake averaged 78 on the four tests. Jenny scored 10 points higher than Blake on the first test, 10 points lower than him on the second test, and 20 points higher on both the third and fourth tests. What is the difference between Jenny's average and Blake's average on these four tets?

8. A square and a triangle have equal perimeters. The lengths of the three sides of the triangle are 6.1 cm, 8.2 cm, and 9.7 cm. What is the area of the square in square centimeters?

9. What is the last digit of $28^{2011} + 9$?

10. Bill walks ½ mile south, then ¾ mile east, and finally ½ mile south. How many miles is he, in a direct line, from his starting point? Express your answer as a fraction reduced to lowest terms.

11. If the largest prime factor of $13! + 15! + x!$ is 421, find the smallest possible value of x.

12. A collection of nickels, dimes and pennies has an average value of 7 cents per coin. If a nickel were replaced by five pennies, the average would drop to 6 cents per coin. How many dimes are in the collection?

13. Adam, Benson, Carrie, David, and Eric leave their identical-looking backpacks in the open cubbies of their school hallway. Unfortunately, a little rascal decides to steal their backpacks and put them on the top of the cubbies. When the five students return to the cubbies, they cannot tell which backpack is which, so they all grab one of the backpacks

at random. What is the probability that all five students grab the wrong backpack? Express your answer as a common fraction.

14. Karen, Peter, and Vignesh are eating a super-large pizza. Karen takes 30 minutes to eat the pizza by herself, Peter takes 20 minutes to eat the pizza by himself, and Vignesh takes 1.5 hours to eat the pizza by himself. Suppose all three of them work together to finish eating the pizza. How long will it take them to do so, if Vignesh gets sick after eating the pizza for 6 minutes, and cannot continue eating any further? Express your answer as a decimal, in minutes.

15. There are three different sizes of juice: small, medium, and large. The medium size costs 50% more than the small size and contains 40% less juice than the large size. The large size contains twice as much juice as the small size and costs 20% more than the medium size. Rank the three sizes from best to worst buy (best = cheapest per volume of juice). Give your answer as an ordered triple, e.g., (S,L,M) where the first letter represents the best buy, and the last the worst buy.

16. There are three consecutive even integers such that the product of the first and the second is 190 greater than the product of -5 and the third integer. What is the value of the largest integer?

17. Find the sum of all distinct elements in the set $\left\{ \frac{1}{m} \middle| m \text{ is a factor of } 2012 \right\}$. By a factor we mean any whole number dividing 2012 including 1 and 2012.

18. Jacob tries to convert a base 10 number to a two digit base 7 but instead accidently switches digits. The first digit would have been 2 greater than the ones place digit if he had converted it correctly. When he converts his result back to base 10, by how much is the result less than his original number?

19. If i is the square root of -1, what is $(5i)\cdot(4i)+21$?

20. In how many ways can a 6 x 6 square with the center 2 x 2 square removed be tiled by 1 x 2 dominoes?

2012 IMSA Junior High Mathematics Competition 7th Team

1. How many sides does a heptagon have?

2. Michael, Jacob, and Bryan each choose a number. The product of Michael and Jacobs' numbers was 24, the product of Michael and Bryans' numbers was 30, and the product of Jacob and Bryans' numbers was 20. What the product of all three numbers?

3. How many ways can we choose 4 elements {1, 2,…12} so that *exactly one* pair of chosen numbers is consecutive?

4. John and Mary are playing a lying game. Each round, each person will make **one statement**: it will be either true or false.

Round	**John**	**Mary**
1	Mary will only tell the truth.	My statement 3 is true.
2	Exactly one of the three statements I make is true.	Exactly 3 statements total will be false.
3	Mary's statement 1 was false.	Exactly one statement this **round** is true.

   How many false statements does John tell?

5. We use $\prod_{k=1}^{n} A_k$ to represent the product of all values of A_k such that k is an integer between l and n, inclusive. For example, $\prod_{k=1}^{4} k = 1 \cdot 2 \cdot 3 \cdot 4 = 24$. What is the value of $\prod_{k=1}^{10} \frac{k}{k+1}$?

6. Let point O be on side $\overline{BC}$ of triangle ABC such that $\overline{OB} = \overline{OA}$. If angle $\angle ABC$ and $\angle ACB$ are complementary, $\overline{AB} = 18$ and $\overline{AC} = 80$, what is the area of triangle, OAC?

7. There is a 25% chance that it will rain on Saturday and, if it does, then there is a 20% chance that it will also rain Sunday. If there is no rain on Saturday, there is a 2/3 chance that it will rain on Sunday. It is Sunday and it is raining. What is the chance that it rained yesterday?

8. You have a quarter, nickel, and dime each made in a different year. The quarter is older than the nickel. What is the probability that the newest coin is the nickel?

9. Find the area of a circle whose circumference is 2.

10. Set P contains all perfect squares with less than 4 digits. Set Q contains all positive perfect cubes with less than 4 digits. What is the sum of all elements in $P \cap Q$?

11. Jose has 15 blue cards and k green cards. If he draws two cards at random without replacement, the probability that the two cards are the same color is equal to the probability that they are different colors. Find all possible values for the number of green cards that Jose has?

12. How many circles of radius 1 may be placed without overlap within a rectangle of dimensions $7 \times (2\sqrt{3}+2)$?

13. Each of the digits 1-4 is used once to create a positive, four-digit integer. What is the probability that the integer formed is divisible by 11? Express your answer as a common fraction in simplest form.

14. How many integers between 1000 and 2012 have all three of the numbers 20, 25, and 30 as factors?

15. In a clock with the hands pointing to 4:40 PM, what is the measure, in degrees, of the acute angle between the two hands of the clock?

16. If 25 square blocks are arranged to make a 5×5, how many different combinations of 3 blocks can be selected so that no two are in the same row or column?

17. Some mathletes at the JHMC are having a car wash to raise money for to attend more math competitions. Initially, 40% of the group are girls. After two girls leave and two boys arrive, 30% of the group are girls. How many girls were initially in the group?

18. Dr. Condie's age is X years; X is also the sum of the ages of his three children. His age Y years ago was three times the sum of their ages then. What is X/Y?

19. If $-6 \le x \le -3$ and $3 \le y \le 6$, what is the largest possible value of $\frac{x+y}{x}$?

20. Consider ordered pairs, (a, b) of positive integers satisfying $ab = 100$. Find the minimum value of

$$\frac{1}{\gcd(a,b)} + \frac{1}{\operatorname{lcm}(a,b)}$$

1. The weatherman predicted that there is a 40% chance of rain tomorrow. What is the probability that it will not rain tomorrow? Express your answer as a percent.

2. What is the area of the region bound by the lines x = 0, y = 0, and y = -8x+4?

3. Abhi is playing hot-shot basketball. His probability of making any free throw (1 point) is 2/5, and his probability of making any 3-pointer is 1/10. Suppose that in one hot-shot game, Abhi decides to shoot 6 free throws and 4 3-pointers. What is his expected score? Express your answer as an exact decimal.

4. James, Kevin, and Lael are three members of a 12-member English class. Their teacher wants to divide the class into two groups of six. Let x be the number of ways that the teacher can do this, if James, Kevin, and Lael must all be in the same group. Find the sum of the digits of x.

5. Find the sum of all prime numbers between 42 and 74.

6. How many circles of radius 1 may be placed without overlap within a rectangle of dimensions $7 \times (2\sqrt{3} + 2)$?

7. Find the units digit of 7^{2012}.

8. Rishi and Srisha are playing a chess match. The first player to win three games wins the match. Rishi has a 3/4 chance of winning any game, while Srisha has a 1/4 chance. Assuming there are no tied games, what is the probability that Rishi wins his chess match against Srisha? Express your answer as a common fraction.

9. You have a semicircle with diameter 8 inches. You decide to cut out two smaller semicircles, each with diameter 4 inches, from the semicircle, with the smaller semicircles' diameters lying on the diameter of the original semicircle. Find the perimeter of the resulting cut-out shape. Express your answer as a decimal rounded to the nearest hundredth.

10. How many lattice points lie inside or on the circle of radius 3 centered at the origin? (A *lattice point* is a point with both coordinates as integers).

11. Let # be the function defined by $a\#b = a^3 + b$. Given $x\#488 = 10^3$, find x.

12. Circular arcs are drawn on the sides of an equilateral triangle with side 10 cm. Each arc has its center at the vertex of the triangle it does not intersect. Find the perimeter of the entire figure formed by the arcs. Express your answer in terms of pi.

13. How many numbers leave a remainder of 12 when 1337 is divided by them?

14. Maggie has 20 identical pieces of Hershey's Kisses. In how many ways can she distribute the candy to 6 of her friends, if her best friend, Evan, must receive at least 2 pieces, and every one of the 6 friends must have at least one piece?

15. Adam can mow a lawn in 3 hours, while Ben can do it in 6 hours, and Carl can do it in 4 hours. If all 3 men work together to mow the lawn, how long will it take them to do so, if Carl gets injured after mowing for 1 hour and cannot continue afterwards? Express your answer as a decimal.

16. Antonio, Wolfgang, Joseph, Johannes, Leonard, Igor, and Ludwig are sitting at a round table, with seats numbered 1 through 7. How many ways can they be seated, if Wolfgang and Ludwig insist on sitting next to each other?

17. If $x+\frac{1}{x}=5$, find $x^2+\frac{1}{x^2}$.

18. Evaluate $1^3 + 2^3 + 3^3 + \ ... + \ n^3$ if n=10.

19. If $x = \sqrt{2+\sqrt{2+\sqrt{2+\cdots}}}$, solve for the value of *x*.

20. Each square of a 4 x 4 grid is filled with a 1, 2, 3, or 4. Define the four *neighbors* of a square *S* to be the squares directly above, to the right, to the left, and below *S*. If a square is located in the top row, its upper neighbor wraps around to the bottom row (like in the figure below, B, D, C, and E are all neighbors of A).

A	B		D
C			
E			

Define $f(a, b)$ as the number of unordered pairs of neighboring squares such that one contains a and the other contains b. It is given that:

$$f(4,1) = 6$$
$$f(4,2) = 3$$
$$f(4,3) = 9$$
$$f(4,4) = 3$$

Find the number of squares containing 4.